SO(10) GUT Model Expectations for θ_{13}

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Predictions for Neutrino Masses and Mixings

have been based on one of the following:

- Grand Unified Models: $SU(5)$, $SO(10)$, $E_6, ...$ which must explain the quark masses and mixings as well;
- Conserved $L_e - L_\mu - L_\tau$ lepton horizontal symmetry;
- Selected texture zeros in the charged lepton L and Dirac N and Majorana M_R neutrino mass matrices;
- Anarchy - presumed absence of symmetry suggests lepton mixing matrix is completely random.

This talk will concentrate on predictions of $SO(10)$ models.

$SO(10)$ **Model Structure**

Essential Ingredients:

- 3 families of 16 LH q's and ℓ's $\to \mathbf{16}_i,\ i = 1, 2, 3$
- Higgs fields in $\{\mathbf{45}_H,\ \mathbf{16}_H,\ \overline{\mathbf{16}}_H\}$ or $\{\overline{\mathbf{126}}_H,\ \mathbf{45}_H,\ \mathbf{54}_H\}$ or $\{\overline{\mathbf{126}}_H,\ \mathbf{120}_H\}$ are needed to break $SO(10) \to$ SM.
- 2 Higgs doublets fit neatly into $\mathbf{10}_H \supset 5 + \bar{5}$ of $SU(5)$ or $\mathbf{10}_H \supset (6, 1, 1) + (1, 2, 2)$ of $SU(4) \times SU(2) \times SU(2)$.
- Doublet-triplet splitting can be achieved via Dimopoulos-Wilczek mechanism, if $\langle \mathbf{45}_H \rangle$ points in $B - L$ direction.
- With only one $\mathbf{10}_H$ effecting the electroweak breaking, $\tan\beta \equiv v_u/v_d \sim 55$.

Additional Higgs fields may be desirable:

- $\mathbf{16}'_H,\ \overline{\mathbf{16}}'_H$ can help to stabilize doublet-triplet splitting.
- If $\langle \bar{5}(\mathbf{16}'_H \rangle \neq 0$, then $H_d \sim \bar{5}(\mathbf{10}_H) \cos\gamma + \bar{5}(\mathbf{16}'_H) \sin\gamma$ and $\tan\beta \sim 1 - 55$ is possible.
- $\mathbf{126}_H$ and $\overline{\mathbf{126}}_H$ are other possibilities.

Additional Matter Fields may be desirable:

- $\mathbf{16},\ \overline{\mathbf{16}}$ pairs may get supermassive and can be integrated out in Froggatt-Nielsen type diagrams for the mass matrix elements.

Horizontal Flavor Symmetries

While $SO(10)$ relates q's and ℓ's of one family, it is necessary to invoke some horizontal flavor symmetry or some effective criterion to avoid the bad $SU(5)$ relations: $m_d = m_e,\ m_s = m_\mu$. This can be done with 4 different levels of model building:

- Level 1: Simply impose a certain texture such as a modified Fritzsch form for the mass matrices.
- Level 2: Introduce an effective λ expansion for each mass matrix. Typically this is done in the context of Froggatt-Nielsen diagrams with a single flavon field carrying some specified flavor charge. The prefactors are not precisely determined, however.
- Level 3: Assign effective operators for each matrix element possibly with some flavor symmetry imposed.
- Level 4: Introduce a horizontal flavor symmetry which assigns flavor charges to every Higgs and matter superfield. Higgs and Yukawa superpotentials are constructed in terms of renormalizable (and possibly some non-renormalizable) terms which obey that flavor symmetry. Matrix elements then follow from Froggatt-Nielsen diagrams.

Types of SO(10) Models

$SO(10)$ models differ by their choice of Higgs structure, horizontal flavor symmetry (if any) and flavor charge assignments.

Two special categories of models exist in literature:

(1) $SO(10)$ Models with Lopsided Textures

- $\mathbf{10}_H,\ \mathbf{16}_H,\ \overline{\mathbf{16}}_H,\ \mathbf{45}_H$ ... Higgs fields are present and couple to matter fields.

- No Higgs representations with rank > 2 are required, but $B - L$ symmetry is broken by 1 unit with $\langle 1(\mathbf{16}_H)\rangle$ and $\langle 1(\overline{\mathbf{16}}_H)\rangle$ VEVs. Hence R-parity is not preserved after the breaking and matter parity must be introduced to retain a stable LSP.

- $H_u \sim \langle 5(\mathbf{10}_H)\rangle$, while H_d arises from a combination of $\langle \bar{5}(\mathbf{10}_H)\rangle$ and $\langle \bar{5}(\overline{\mathbf{16}'}_H)\rangle$.

- With a flavor symmetry present, lopsided D and L mass matrices result and moderate values of $\tan\beta$ are possible.

- Large $U_{\mu 3}$ atmospheric neutrino mixing but small V_{cb} quark mixing result from this lopsided structure.

- Somewhat enhanced leptonic flavor violation is also predicted in $\mu \to e + \gamma$ and $\tau \to \mu + \gamma$.

(2) Minimal $SO(10)$ Models with Symmetric Texture

- $\mathbf{10}_H$, $\overline{\mathbf{126}}_H$ are only EW symmetry-breaking Higgs coupled directly to the matter fields.

- $\mathbf{120}_H$ or $\mathbf{45}_H + \mathbf{54}_H$ needed to break $SO(10)$ to the SM.

- Tensor representations of rank > 2 are disfavored by string theory, but R-parity is preserved after $B - L$ symmetry is broken by 2 units with a $\langle(10,1,3)\overline{\mathbf{126}}_H\rangle$ VEV.

- H_u, H_d Higgses are combinations of doublets in $\mathbf{10}_H$, $\overline{\mathbf{126}}_H$, so moderate values of $\tan\beta$ are possible.

- Flavor symmetry is not required as linear combinations of $\mathbf{10}_H$, $\overline{\mathbf{126}}_H$ VEVs with known Clebsches for the mass matrix elements can be directly determined by mass and mixing data. This corresponds to an effective operator approach.

Type I canonical seesaw mechanism involves only the Dirac and right-handed Majorana neutrino mass matrices:

$$M_\nu = -N^T M_R^{-1} N.$$

Type II seesaw also includes the left-handed Majorana mass matrix involving an induced triplet VEV which can arise if both parity and $B - L$ are broken at the same scale:

$$M_\nu = M_L - N^T M_R^{-1} N.$$

With Type II seesaw and induced left-handed Majorana term dominant, large atmospheric neutrino mixing follows from $b-\tau$ unification.

Selected $SO(10)$ Models and θ_{13} Predictions

Model	(Level) Flavor Sym.	Texture	$\tan\beta$	$\sin^2 2\theta_{13}$
AB	(4) $U(1) \times Z_2 \times Z_2$	Lopsided	~ 5	0.0008-0.003
ABMSV	(1) Min. eff. ops.	Sym (II)	?	0.10
BO	(1) Min. eff. ops.	Sym	?	0.004-0.008
BKOT	(1) Min. eff. ops.	Sym	?	0.0004-0.01
BPW	(3) $U(1)$ eff. ops.	Sym (II)	low	?
CM	(4) $U(2) \times (Z_2)^3$	Sym	10	0.09
FKO	(1) Min. eff. ops.	Sym	45	0.16
GMN	(1) Min. eff. ops.	Sym(II)	10	0.10
KM	(2) $SU(3) \times U(1)$	Lopsided	small	~ 0.19
KRV-S	(4) $SU(3) \times Z_2 \times U(1)_A$	Sym/Asym	?	0.02
M	(2) $U(1)_A \times Z_2$	Lopsided	5	~ 0.19

AB	Albright, Barr	$\sin^2 2\theta_{atm} \simeq 0.99$
ABMSV	Aulakh, Bajc, Melfo, Senjanovic, Vissani	not spelled out
BO	Bando, Obara	
BKOT	Bando, Kaneko, Obaro, Tanimoto	
BPW	Babu, Pati, Wilczek	
CM	Chen, Mahanthappa	$\sin 2\beta = 0.74$, $\delta_{CKM} \sim 35°$
FKO	Fukuyama, Kikuchi, Okada	$\Delta m^2_{sol}/\Delta m^2_{atm} = 0.188$
GMN	Goh, Mohapatra, Ng	$\sin^2 2\theta_{atm} \lesssim 0.92$, $\sin^2 2\theta_{12} \gtrsim 0.9$
KM	Kitano, Mimura	satisfies LMA mixing?
KRV-S	King, Ross, Velasco-Sevilla	
M	Maekawa	satisfies LMA mixing?

General Observations

Taking into account the revised Super-K value of $\Delta m^2_{32} \simeq 0.20 \times 10^{-3}$ eV^2, the present 3σ CHOOZ limit implies $\sin^2 2\theta_{13} \lesssim 0.27$. Increased statistics and new reactor data as well as data from off-axis detectors should be able to reduce the CHOOZ limit down to $\sin^2 2\theta_{13} \lesssim 0.02$ and negate or confirm the predictions indicated previously in red.

The follow general observations follow from the sample of models illustrated.

- All the $SO(10)$ models considered lead to a normal hierarchy for the light neutrino masses. A partial degeneracy of the lighter two masses is possible in the Type II seesaw models. This is to be contrasted with the Zee- or conserved lepton-type models which favor an inverted hierarchy.
- The models preferring the larger allowed values for $\sin\theta_{13}$ tend to have symmetric textures though there are several exceptions.
- The lopsided models are fewer in number, and no general conclusion can be drawn.
- With bimaximal mixing and $\sin\theta_{13} = 0$ at the GUT scale, radiative corrections can magnify the value to $\sin\theta_{13} \sim 0.01$ at the EW scale for quasi-degenerate neutrinos, while the correction is negligible for a normal hierarchical spectrum.
- For those models with the smallest values of $\sin\theta_{13}$, Superbeams or Neutrino Factories may be required to check their predictions.

CP Violation in $SO(10)$ Models

The interesting small unknown parameter in the MNS mixing matrix is

$$U_{e3} = e^{-i\delta} \sin\theta_{13},$$

where δ is the Dirac CP-violating phase.

Two CP-violating Majorana phases appear in a diagonal phase matrix multiplying U_{MNS} from the right:

$$\begin{aligned} V_{MNS} &\equiv U_L^\dagger U_\nu = U_{MNS}\Phi \\ &= \begin{pmatrix} c_{12}c_{13} & c_{13}s_{12} & e^{-i\delta}s_{13} \\ -\ldots & +\ldots & c_{13}s_{23} \\ +\ldots & -\ldots & c_{13}c_{23} \end{pmatrix} diag(e^{i\chi_1}, e^{i\chi_2}, 1). \end{aligned}$$

This comes about because the charged lepton mass matrix L is diagonalized by a left-handed unitary transformation U_L so that

$$U_L^\dagger (L^\dagger L) U_L = diag(m_e^2,\ m_\mu^2,\ m_\tau^2)$$

where U_L transforms $L^\dagger L$ from the $SO(10)$ flavor basis to the mass basis. An arbitrary phase transformation can be made on U_L.

The left-handed complex symmetric light neutrino matrix M_ν obtained from the seesaw mechanism is transformed by

$$U_\nu^T M_\nu U_\nu = diag(m_1,\ m_2,\ m_3).$$

In this case the phase is fixed in U_ν in order to give positive, real mass eigenvalues. Hence V_{MNS} can only be brought into the standard U_{MNS} form by a phase transformation from the left. Any necessary phase transformation on the right of U_{MNS} must be undone by the phase matrix Φ.

If $\sin\theta_{13}$ turns out to be large enough, the Dirac δ can be determined in oscillation experiments with reactor and off-axis long baseline neutrino beams. The Majorana phases play a role in neutrino-less double beta decay experiments and leptogenesis.

Determination of these phases will place even more constraints on $SO(10)$ model building. Unfortunately, many models are not accurate or detailed enough to make predictions for these phases. But they are important.

For example, in the AB model the mass and mixing data were found to be well satisfied with a real right-handed Majorana matrix M_R. This led to a very small $\delta \sim 5^\circ$ and no leptogenesis. But by making M_R complex, the model can yield a large resonant enhancement of the lepton asymmetry and a Dirac phase closer to 90°. Actually in order to maintain the good quark and lepton mass and mixing predictions, the lepton asymmetry appears to be constrained, so that the baryogenesis parameter which should be $\eta_B = 6.3 \times 10^{-10}$ falls short in this model by a factor of 5 or so.

Summary

- A number of $SO(10)$ SUSY GUT models have been proposed in the literature which are presently viable and yield the LMA solar neutrino solution.

- Long baseline experiments which can determine whether the neutrino mass hierarchy is normal or inverted appear to have a direct bearing on the survival of $SO(10)$ vs conserved-lepton-number-type models.

- The observed value of $\sin\theta_{13}$ will further narrow down the list of viable models. Some predict that θ_{13} lies just below the CHOOZ bound and will be observable with reactors and/or off-axis beams. Others favor such low values of θ_{13} that Superbeams and/or Neutrino Factories may be required to determine its value.

- Determination of the Dirac CP phase will be possible with reactors and off-axis beams if θ_{13} is large enough and will further distinguish acceptable models.

- The two Majorana CP phases are typically beyond reach in neutrino-less double beta decay in $SO(10)$ models with normal hierarchy, but they can play an important role in leptogenesis which is a presently popular ingredient for baryogenesis in the Universe.